SPIN-2000/07
hep-th/0003005

Determinism and Dissipation in Quantum Gravity*

Gerard 't Hooft

Institute for Theoretical Physics
University of Utrecht, Princetonplein 5
3584 CC Utrecht, the Netherlands

and

Spinoza Institute
Postbox 80.195
3508 TD Utrecht, the Netherlands

e-mail: g.thooft@phys.uu.nl
internet: http://www.phys.uu.nl/~thooft/

Abstract

Without invalidating quantum mechanics as a principle underlying the dynamics of a fundamental theory, it is possible to ask for even more basic dynamical laws that may yield quantum mechanics as the machinery needed for its statistical analysis. In conventional systems such as the Standard model for quarks and leptons, this would lead to hidden variable theories, which are known to be plagued by problems such as non-locality. But Planck scale physics is so different from field theories in some flat background space-time that here the converse may be the case: we speculate that causality and locality can only be restored by postulating a deterministic underlying theory. A price to be paid may be that the underlying theory exhibits dissipation of information.

* presented at "Basics and Highlights in Fundamental Physics", Erice, August, 1999.

1. Introduction

In the opening lecture of this School [1] it was explained that the physical degrees of freedom of a black hole are distributed on its horizon in such a way that there appears to be one Boolean degree of freedom per unit of surface area of size

$$A_0 = 4 \ln 2 \, L^2_{\rm Planck} \, . \tag{1.1}$$

These are the degrees of freedom that appear in the equations of motion, c.q. the Schrödinger equation for a black hole. In turn, according to the principles of General Relativity, these same equations of motion should apply to what happens in the (nearly) flat space-time experienced by an in-going observer. This led us to the formulation of "the Holographic Principle", according to which the physical degrees of freedom that describe all physical events in some definite region of space and time, can be mapped onto a two-dimensional surface in such a way that there is exactly one Boolean degree of freedom per unit of surface of size A_0.

One striking aspect of this observation is that the same region of space-time could be used to describe a different black hole with its horizon some place else. Apparently, one may choose any member of an infinite set of possible surfaces to project the physical degrees of freedom onto it. Also, one should be able to map directly from one surface onto another.

It now appears to be a satisfactory feature of string theories [2] and certain semi-perturbative extensions of them, that they manage to reproduce this holographic principle [3]. This principle implies that any complete theory combining quantum mechanics with gravity should exhibit an upper limit to the total number of independent quantum states that increases exponentially with the surface area of a system, rather than its volume.

All this raises a number of important questions. First, what does *locality* mean for such theories? And how can notions such as causality, unitarity, and local Lorentz invariance make sense if there is no trace of 'locality' left? In this lecture, a theory is developed that will *not* postulate the quantum states as being its central starting point, but rather classical, deterministic degrees of freedom [4]. Quantum states, being mere mathematical devices enabling physicists to make statistical predictions, may turn out to be derived concepts, with a not strictly locally formulated definition. Once it is realized that quantum states may be non-local, derived concepts, it is natural to consider making one more step.

In the past, many versions of hidden variable theories were dismissed by a majority of researchers for two reasons: one reason was that these theories did not seem to work properly in the sense that counter examples could be constructed using eigenstates of certain symmetries: rotation symmetry, isospin symmetry, and so on. The second reason was that there appeared to be no need for such theories.

We observe that most of the familiar symmetries are absent at the Planck scale. There are clearly no conservation laws such as isospin, and, in the absence of true locality one cannot rotate any system with respect to a reference system, since they do not decouple. Constructing counter examples to hidden variable theories then becomes a lot harder. Also we claim that some relaxed version of quantum mechanics could be extremely helpful in bypassing the holographic principle; locality could be restored, and once again causality could be reconciled with (a weaker version of) local Lorentz invariance, or general coordinate invariance. We suspect that such steps may be needed in constructing logically coherent theories for Planck scale physics.

Thus, one may have one or several motivations for a reconsideration of "hidden variable" theories:

i. Einstein's wish for "reality". At the time this is written, the quantum mechanical doctrine, according to which all physical states form a Hilbert space and are controlled by non-commuting operators, is fully taken for granted in theoretical physics. No return to a more deterministic description of "reality" is

considered necessary; to the contrary, string theorists often give air to their suspicion that the real world is even crazier than quantum mechanics. One might however complain that the description of what really constitutes concepts such as space, time, matter, causality, and the like, is becoming increasingly and uncomfortably obscure. By some physicists this may be regarded as an inescapable course of events, with which we shall have to learn to live, but others such as this author strongly prefer a more complete description of the notion of reality. We admit however that at scales relevant to atomic physics, or even the Standard Model, there is no direct logical inconsistency to be found in Quantum Mechanics.

ii. "Quantum Cosmology". How would one describe the 'wave function of the universe'? An extremely important example of a quantum cosmological model, is a model of gravitating particles in 1 time, 2 space dimensions [5]. Here, a complete formalism for the quantum version at first sight seems to be straightforward [6], but when it comes to specifying exact details, one discovers that we cannot rigorously define what quantum mechanical amplitudes are, what it means when it is claimed that "the universe will collapse with such-and-such probability", what and where the observers are, what they are made of, and so on. Eventually, one would have to admit that at cosmological scales, any experiment is done only once, yielding answers either 'yes' or 'no', but never probability distributions. The cosmological wave function is a dubious notion. Quite conceivably, quantum mechanics as we know it only refers to repeatable experiments at small time and distance scales. The true laws of physics are about certainties, not probabilities.

Note that, since the entire hamiltonian of the universe is exactly conserved, the "wave function of the universe" would be in an exact eigenstate of the hamiltonian, and therefore, the usual Schrödinger equation is less appropriate than the description of the evolution in the so-called Heisenberg representation. Quantum states are space-time independent, but operators may depend on space-time points — although only if the location of these space-time points can be defined in a coordinate-free manner![†]

iii. Even at a local scale (i.e. not cosmological), there are problems that we could attribute to a clash with Quantum Mechanics. Apart from the question of the cosmological principle, these are:

- the non-renormalizability of gravity;
- the fact that the gravitational action (the Einstein-Hilbert action) is not properly bounded in Euclidean space, while the Maxwell and Yang-Mills actions are. This is related to the fundamental instability of the gravitational force.
- topologically non-trivial quantum fluctuations. They could destroy the causal coherence of any theory. Perhaps most such fluctuations may have to be outlawed, as they would also require the boundary conditions to fluctuate into topologically non-trivial ones.
- black holes cause the most compelling conflicts with local quantum mechanics.
- there still is the mystery of the cosmological constant. it appears to require a reconsideration not only of physical principles at the Planck scale, but also at cosmological scales, since we are dealing here with an infrared divergence that appears to be cancelled out in a way that requires new physics.

[†] Note that, besides energy, also total momentum and angular momentum of the universe must be conserved (and they too must be zero).

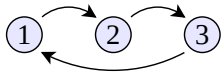
2. States and probabilities

Imagine a universe in which different "states" are possible. A state may be characterized any possible way:

$$|\psi\rangle \;=\; \Big|\, \vec{x}_1,\ \vec{\sigma}_1,\ \raisebox{-0.5em}{\includegraphics[height=3.5em]{figure.pdf}}\ ,\ \begin{matrix}\text{your name and}\\ \text{telephone}\\ \text{number}\end{matrix} \,\Big\rangle\ , \tag{2.1}$$

There are two ways in which we can introduce the equations of motion, and what we mean by 'state' depends on that. At first sight it seems most natural to postulate that a state evolves in time. So, at a given time t we have a state, and there is some equation that tells us what this state looks like at later (or maybe also at earlier) times t'. We refer to this as the Schrödinger picture. But when we realize that the notion of time may depend on the clocks used, or more generally, that it requires the introduction of Cauchy surfaces, we might opt for a different notion of "state". A state is then defined as time-independent. The universe is in a particular state, and in this state all observables may depend on time; the time variable is then linked to the observable, $\mathcal{O}(t)$. This is the Heisenberg picture. It is familiar from quantum mechanics, where now observables are said to be time-dependent, obeying an evolution equation of the form $\mathrm{d}\mathcal{O}(t)/\mathrm{d}t = -i[\mathcal{O}(t),\, H]$. We will frequently switch between Schrödinger and Heisenberg picture.

Let us begin with the Schrödinger picture. For simplicity then we take time to be discrete (in the Heisenberg picture, the question whether time is discrete or continuous would not be so important). A simple example is a universe that can only be in three different states, and we have a prescription for the evolution as follows:



This of course is a completely deterministic universe. Nevertheless, it may be useful to introduce the Hilbert space spanned by these three states, so as to enable us to handle the evolution statistically. In this space, the one-time-step evolution operator would be

$$U = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\ , \tag{2.2}$$

a unitary matrix. Suppose we also consider states of the form

$$|\psi\rangle = \alpha|1\rangle + \beta|2\rangle + c|3\rangle\,, \tag{2.3}$$

then after one time step, we would have

$$|\psi\rangle_{t+1} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} |\psi\rangle_t = U(t,t+1)|\psi\rangle_t\,, \tag{2.4}$$

We may *define* the probability $P(i)$ for being in a state $|i\rangle$ as

$$P(1) = |\alpha|^2\ ; \quad P(2) = |\beta|^2\ ; \quad P(3) = |\gamma|^2\ , \tag{2.5}$$

and observe that probability conservation corresponds to unitarity of the evolution matrix U.

Let us now turn to a basis in which U is diagonal:

$$U \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{2\pi/3} & 0 \\ 0 & 0 & e^{-2\pi/3} \end{pmatrix}\ . \tag{2.6}$$

One then may write

$$U = \exp\big(-iH\delta t\big)\ , \tag{2.7}$$

where the unit time interval δt will often be taken to be one. So, we can take as our 'hamiltonian':

$$H = \begin{pmatrix} 0 & & \\ & -2\pi/3 & \\ & & +2\pi/3 \end{pmatrix}\ . \tag{2.8}$$

Note that this is the hamiltonian of an atom with a magnetic moment in a homogeneous magnetic field, although it should of course also be noted that all eigenvalues of H are only well-defined *modulo* integral multiples of 2π.

All that was needed for the above manipulations is that the deterministic law used as a starting point was time-reversible, otherwise the matrix U would not have been unitary. Thus, we make our first important observation: [7] *any deterministic, time-reversible system can be described using a quantum mechanical Hilbert space, where states obey a Schrödinger equation, and where the absolute squares of the coefficients of the wave functions represent probabilities.* Prototype examples are clocks that count periodically over N different states — they can be mapped onto atoms with spin j in a magnetic field, $N = 2j+1$.

The converse is not true: most simple quantum mechanical systems do not allow a deterministic interpretation. If there would only be a finite and small number of states, it is rather easy to read off when a deterministic interpretation may be allowed, but in an infinite volume limit this is not at all very straightforward, in particular when non-local transformations are allowed, *as seems to be the case in gravitational physics such as strings and black holes!* Can a mapping condition be formulated?

If, in a quantum theory, in its Heisenberg formulation, a complete set of operators $\mathcal{O}(t)$ can be found that mutually commute at all times,

$$[\mathcal{O}(t),\, \mathcal{O}(t')] = 0\ , \quad \forall\,(t,\, t')\ , \tag{2.9}$$

then the theory may be said to be deterministic. A set of operators is complete if any given set of eigenvalues $O(t)$ unambiguously specifies a basis element of Hilbert space. We may take such operators to define expectation values of 'truly existing' observables. In honor of J. Bell, we call these operators 'beables'. The basis generated this way will be called a *primordial* basis. It could be that different complete sets of beables can be found, one set not commuting with another, so that we have different choices for the primordial basis. In that case we will have several competing 'theories' for the ontological facts described by our equations.

Operators P that do not commute with the beables of the theory, such as the evolution operator $U(t,t')$ of Eq. (2.2), will be called 'changeables'.

Quantum physicists in search of a deterministic theory have the assignment: find a complete set of beables for the Standard Model, or a modified version of it. Equivalently, find a primordial basis. In the model of an atom spinning in a magnetic field, we succeeded in doing just that. The beables are the matrices diagonal in the primordial basis $\big\{|1\rangle, |2\rangle, |3\rangle\big\}$ of Eq. (2.3).

3. 'Neutrinos'

There is a more interesting system, actually realized to some extent in the real world, for which a primordial basis can be constructed. [4] Consider massless, non-interacting chiral fermions in four space-time dimensions. We can think of neutrinos, although of course real neutrinos deviate slightly from the ideal model described here.

First, take the first-quantized theory. The hamiltonian for a Dirac particle is

$$H = \vec{\alpha}\cdot\vec{p} + \beta m\,, \qquad \{\alpha_i,\,\alpha_j\} = 2\delta_{ij}\,, \quad \{\alpha_i,\,\beta\} = 0\,, \quad \beta^2 = 1\,. \tag{3.1}$$

Taking $m = 0$, we can limit ourselves to the subspace projected out by the operator $\frac{1}{2}(1+\gamma_5)$, at which point the Dirac matrices become two-dimensional. The Dirac equation then reads

$$H = \vec{\sigma}\cdot\vec{p}, \tag{3.2}$$

where $\sigma_{1,\,2,\,3}$ are the Pauli matrices. We now consider the following candidates for 'beables':

$$\left\{\,\hat{p}\,, \quad \hat{p}\cdot\vec{\sigma}\,, \quad \hat{p}\cdot\vec{x}(t)\,\right\}\,, \tag{3.3}$$

where $\hat{p}$ stands for $\pm\vec{p}/|p|$, with the sign such that $\hat{p}_x > 0$. We do *not* directly specify the sign of $\vec{p}$.

Writing $p_j = -i\frac{\partial}{\partial x_j}$, one readily checks that these three operators commute, and that they continue to do so at all times. Indeed, the first two are constants of the motion, whereas the last one evolves into

$$\hat{p}\cdot\vec{x}(t) = \hat{p}\cdot\vec{x}(0) + \hat{p}\cdot\vec{\sigma}\,t\,. \tag{3.4}$$

The fact that these operators form a complete set is also easy to verify: in momentum space, $\hat{p}$ determines the orientation; let us take this to be the z direction. Then, in momentum space, the absolute value of p, as well as its sign, are identified with its z-component, and it is governed by the operator $i\partial/\partial p_z = x_z = \hat{p}\cdot\vec{x}$. The spin is defined in the z-direction by $\hat{p}\cdot\vec{\sigma}$.

Mathematically, these equations appear to describe a *plane*, or a flat membrane, moving in orthogonal direction with the speed of light. Given the orientation (without its sign) $\hat{p}$, the coordinate $\hat{p}\cdot\vec{x}$ describes its distance from the origin, and the variable $\hat{p}\cdot\vec{\sigma}$ specifies in which of the two possible orthogonal directions the membrane is moving. Note that, indeed, this operator flips sign under 180° rotations, as it is required for a spin $\frac{1}{2}$ representation. This, one could argue, is what a neutrino really is: a flat membrane moving in the orthogonal direction with the speed of light. But we'll return to that later: the theory can be further improved (see the end of Sect. 4).

We do note, of course, that in the description of a single neutrino, the hamiltonian is not bounded from below, as one would require. In this very special model, there is a remedy to this, and it is precisely Dirac's second quantization procedure. We consider a space with an infinite number of these membranes, running in all of the infinitely many possible directions $\hat{p}\cdot\vec{\sigma}$. In order to get the situation under control, we introduce a temporary cut-off: in each of the infinitely many possible directions $\hat{p}$, we assume that the membranes sit in a discrete lattice of possible positions. The lattice length a may be as small as we please. Furthermore, consider a box with length L, being as large as we please. The first-quantized neutrino then has a finite number of energy levels, between $-\pi/a$ and $+\pi/a$. The state we call 'vacuum state', has all negative energy levels filled and all positive energy levels empty. All excited states now have positive energy. Since the Dirac

particles do not interact, their numbers are exactly conserved, and the collection of all observables (3.3) for all Dirac particles still correspond to mutually commuting operators.

In this very special model we thus succeed in producing a complete primordial basis, generated by operators that commute with one another at all times (beables), whereas the hamiltonian is bounded from below. We consider this to be an existence proof, but it would be more satisfying if we could have produced a less trivial model. Unfortunately, our representation of neutrinos as infinite, strictly flat membranes, appears to be impossible to generalize so as to introduce mass terms and/or interactions.

Further attempts at obtaining more realistic models which are fundamentally quantum mechanical yet allow for a deterministic interpretation failed because it did not appear to be possible to create a hamiltonian that is bounded below, so that a very special state can be selected, being the lowest eigenstate, which can be identified unambiguously as the vacuum state. In the Schrödinger picture, two classes of models may be considered: the ones with *continuous time*, and the ones with *discrete time*. If time is continuous, and a set of beables $q_i(t)$ is found, a natural choice for the hamiltonian would be

$$H = \sum_i p_i \cdot f_i(\mathbf{q}) \ ; \quad \dot{q}_i = f_i(\mathbf{q}) \ , \tag{3.5}$$

where p_i are the ordinary momentum operators associated to q_i. We see immediately that, in spite of the quantum mechanical notation, the q_i evolve in a deterministic manner. But we also see that, regardless the choice for the functions f_i, this hamiltonian can never be bounded below.

Time does not have to be continuous. It would suffice if a set of beables could be defined to form a sufficiently dense lattice in space-time, and this brings us to cellular automaton models. In these models, only a finite number of possible states is needed in a given spacelike volume. One might hope that then also the Hamilton density would be a finite-dimensional matrix, so that the existence of a ground state might follow, in certain cases. Again, however, there is a problem. Let now

$$q_i(t+1) = f_i\big(\mathbf{q}(t)\big) \, , \tag{3.6}$$

then this defines uniquely the evolution operator over integer time steps:

$$U(t+1,t) = e^{-iH} \, , \tag{3.7}$$

but then this defines the eigenvalues of H only *modulo* 2π. Again, the notion of a lowest eigenstate is questionable.

In the Heisenberg picture, the dimensionality of a limit cycle does not change if we replace the time variable by one with smaller time steps, or even a continuous time. Working with a continuous time variable then has the advantage that the associated operator, the hamiltonian, is unambiguous in that case.

The neutrino example of this section does show that in some cases a bounded hamiltonian may yet exist (here, it is the continuum limit that singles out a special choice for H with unambiguous eigenvalues), so one may hope that more is possible, but something drastically new may be needed.

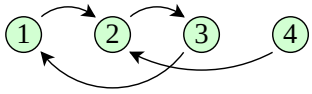


Fig. 1. Transition rule with information loss.

4. Information loss

The new ingredient needed might be information loss [8]. At first sight this is surprising. One would have thought that, with information loss, the evolution operator will no longer be unitary, and hence no quantum mechanical interpretation is allowed. Consider however a model universe with 4 elements. They will be indicated not by Dirac brackets, but as (1), (2), (3) and (4). The evolution law is as depicted in Fig. 1.

If we would associate basis elements of a Hilbert space to each of these states, the evolution operator would come out as

$$U(t+1,t) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} , \tag{4.1}$$

and this would not be a unitary operator. Of course, the reason why the operator is not unitary is that the evolution rule (2.1) is not time reversible. After a short lapse of time, only states (1), (2) and (3) can be reached. In this simple example, it is clear that one should simply erase state (4), and treat the upper left 3×3 part of Eq. (4.1) as the unitary evolution matrix. Thus, the quantum system corresponding to the new evolution law is three-dimensional, not four-dimensional, and it seems to be trivial to restore time-reversibility and unitarity.

In more complicated non-time-reversible evolving systems, however, the 'genuine' quantum states and the false ones (the ones that cannot be reached from the far past) are actually quite difficult to distinguish. For this reason, we introduce the notion of *equivalence classes*. Two states are called equivalent if, after some finite time interval, they evolve into the same state. The system described above has three equivalence classes,

$$E_1 = \{(1),\,(4)\}\ , \quad E_2 = \{(2)\}\ , \quad E_3 = \{(3)\}\,. \tag{4.2}$$

Quantum states will now be identified with these equivalence classes. In our example, in the Schrödinger picture, we have three basis elements, $|1\rangle = E_1;\ |2\rangle = E_2;\ |3\rangle = E_3$. *In terms of these objects* $E_1,\ E_2,\ E_3$, one has an evolution operator

$$U = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} , \tag{4.3}$$

and a hamiltonian operator H can be defined such that $U = e^{-iH}$. Our model universe would be in an eigenstate of this hamiltonian.

An extreme example of a situation where equivalence classes must occur is the black hole. We imagine a theory with classical (that is, not quantum mechanical) general relativistic field equations. Classical black holes may result as solutions. Since this classical system surely has information loss, the equivalence classes E_i will each comprise all possible initial states that result in the same black hole (with the same mass, charge and angular momentum) after collapse. It is important to realize that the equivalence classes may be much smaller than the classes of primordial 'states'. Their definition is not local, in the sense that two states that differ at different locations may belong to the same equivalence class. One might suspect that this could explain the apparent need for non-locality in conventional attempts at constructing hidden variable theories.

The physical distinction between theories with information loss and theories without information loss is not very clear in models with a small number of distinct states such as the example given above. After all, one may simply ignore the 'unreachable' states and notice that the universe ends up in a limit cycle that is indistinguishable from what a universe without information loss would do. It is the fact that the true universe is far too large ever to end up in a limit cycle, and the fact that we wish to understand small regions of this universe, nearly but no quite decoupled from the rest of the world, which make the introduction of information loss and equivalence classes non-trivial. The most acute problem to be addressed is how to create a hamiltonian that can be viewed as the integral over a hamiltonian density that is bounded below and has (more or less) local commutation rules.

What we address next is the question how to construct the laws of physics for a universe that is essentially open, *i.e.* it consists of smaller parts glued together. These smaller parts form an infinite 3 (space-)dimensional world. We begin with gluing two small pieces together.

If left alone, a tiny segment of the universe may be assumed to enter into a limit cycle. In fact, it may have several different cycles to choose from, depending on the initial state. In the Schrödinger picture, each of these cycles forms a sequence of a large number of states, and there are numerous different energy eigenstates that the system can choose from. But, as stated before, if time is not an extrinsically defined coordinate, it is meaningless to consider time dependence. We then prefer the Heisenberg picture, where we have only one state for each cycle. In this state it is the observables that take a periodic sequence of values, but only if time could be defined at all. In any case, the dimensionality of Hilbert space is then determined by the number of different possible cycles.

If we have two adjacent segments of the universe, the *relative* time coordinate is well-defined and important. It may or may not be defined as an arbitrary real number. A way to introduce coupling is as follows. We introduce a cyclic time coordinate on each segment (which could be seen as a reintroduction of the Schrödinger picture). Now assume that the relative speed of the time evolution of the two adjacent segments is determined by local dynamics. Using general relativistic notation, one would have

$$\frac{\mathrm{d}}{\mathrm{d}t}\big(|\psi_1\rangle|\psi_2\rangle\big) = \Big(\sqrt{g^{00}(1)}\,\frac{\partial}{\partial t}|\psi_1\rangle\Big)|\psi_2\rangle + \sqrt{g^{00}(2)}\;|\psi_1\rangle\frac{\partial}{\partial t}|\psi_2\rangle\,. \tag{4.4}$$

Here, $\sqrt{g^{00}}$ may be interpreted as the 'gravitational potential field', and the ratio $\sqrt{g^{00}(2)}/\sqrt{g^{00}(1)}$ is the 'gravitational field strength'.

Take the case that each segment has just one limit cycle. Each now has a periodic 'time' variable $q \in [\,0,\,1\,)$, but we also take an external time variable t, so we have $q_1(t)$ and $q_2(t)$. Let the undisturbed time derivatives be

$$\mathrm{d}q_i/\mathrm{d}t \equiv \dot{q}_i(t) = v_i\,, \tag{4.5}$$

so that the (undisturbed) periods are $T_i = 1/v_i$. The really relevant quantity is the ratio

$$\mathrm{d}q_1(t)/\mathrm{d}t : \mathrm{d}q_2(t)/\mathrm{d}t = \mathrm{d}q_1/\mathrm{d}q_2\ .$$

In Fig. 2, this is the slope of the trajectories for the solutions.

The Schrödinger Hilbert space is spanned by the states $|q_1,\,q_2\rangle$, and our formal hamiltonian is

$$H = v_1p_1 + v_2p_2\,; \qquad p_j = -i\partial/\partial q_j\,. \tag{4.6}$$

In this case, even the zero-energy states span an infinite Hilbert space, so, in the Heisenberg picture, the product universe has an infinity of possible states.

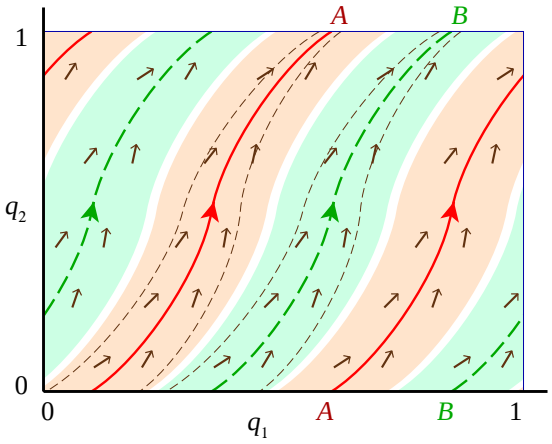


Fig. 2. Flow chart of a continuum model with two periodic variables, q_1 and q_2. In this example, there are two stable limit cycles, A and B, representing the two 'quantum states' of this 'universe'. In between, there are two orbits that would be stable in the time-reversed model.

Information loss is now introduced by adding a tiny perturbation that turns the flow equations into a non-Jacobian one:

$$v_1 \to v_1^0 + \varepsilon f(q_1, q_2)\,; \qquad v_2 \to v_2^0 + \varepsilon g(q_1, q_2)\,. \tag{4.7}$$

The effect of these extra terms can vary a lot, but in the generic case, one expects the following (assuming ε to be a sufficiently tiny number):

Let the ratio v_1^0/v_2^0 be sufficiently close to a rational number N_1/N_2. Then, at specially chosen initial conditions there may be periodic orbits, with period

$$P = v_1^0/N_1 = v_2^0/N_2\,, \tag{4.8}$$

where now v_1^0 and v_2^0 have been tuned to exactly match the rational ratio — possible deviations are absorbed into the perturbation terms. Nearby these stable orbits, there are non-periodic orbits, which in general will converge into any one of the stable ones, see Fig. 2. After a sufficiently large lapse of time, we will always be in one of the stable orbits, and all information concerning the extent to which the initial state did depart from the stable orbit, is washed out. Of course, this only happens if the Jacobian of the evolution, the quantity $\sum_i (\partial/\partial q_i)\dot{q}_i$, departs from unity. Information loss of this sort normally does not occur in ordinary particle physics, although of course it is commonplace in macroscopic physics, such as the flow of liquids with viscosity (see Sect. 6).

The stable orbits now represent our new equivalence classes (note that, under time reversal, there are other stable orbits in between the previous ones). Most importantly, we find that the equivalence classes will form a discrete set, in a model of this sort, most often just a finite set, so that, back in the Heisenberg picture, our 'universe' will be just in a finite number of distinct quantum states.

Generalizing this model to the case of more than two periodic degrees of freedom is straightforward. We see that, if the flow equations are allowed to be sufficiently generic (no constraints anywhere on the values of the Jacobians), then distinct stable limit orbits will arise. There is only one parameter that remains continuous, which is the global time coordinate. If we insert $H|\psi\rangle = 0$ for the entire universe, then the global time coordinate is no longer physically meaningful, as it obtains the status of an unobservable gauge degree of freedom.

In the above models, what we call 'quantum states', coincides with Poincaré limit cycles of the universe. We repeat, just because our model universes are so small, we were able to identify these. When we glue tiny universes together to obtain larger and hence more interesting models, we get much longer Poincaré cycles, but also much more of them. Eventually, in practice, sooner or later, one has to abandon the hope of describing complete Poincaré cycles, and replace them by the more practical definitions of equivalence classes. At that point, when one combines mututally weakly interacting universes, the effective quantum states are just multiplied into product Hilbert spaces.

Our introduction of information loss and equivalence classes sheds new light on the neutrino model introduced in Sect. 3. In that section, we concluded that neutrinos seem to 'be' infinite, flat, sheets, which may eventually become untenable when space-time curvature and other interactions or mass terms are taken into account. Now, we have another option. The sheets are not the primordial beables but they are the equivalence classes. We could have that neutrinos are more-or-less conventional, but classical, point particles, with auxiliary velocity vectors $\mathbf{p}$ of unit length. The rules of motion are now such that the velocity in the direction of $\mathbf{p}$ is rigorously fixed to be c, but the velocities in the transverse directions are chaotic and not re-traceable due to information loss. In such a picture it is no longer impossible to imagine tiny deviations from the rule to incorporate interactions and tiny mass terms.

5. Harmonic and anharmonic oscillators.

What we have so far is a strategy. We still have the question how a model, either with or without information loss, can emerge in such a way that the total hamiltonian is the integral of a Hamilton density bounded from below, so that we can understand the chaotic nature of our vacuum. The neutrino model was one very special case. Now let us concentrate on the most elementary building block for bosonic fields in Nature: the harmonic oscillator, with possible disturbances.

A *classical* harmonic oscillator may be described by the equations

$$\dot{x} = y \ ; \quad \dot{y} = -x \ . \tag{5.1}$$

The (Schrödinger) states are then all sets $|x,\, y\rangle$, and the classical equation (5.1) is generated by the 'hamiltonian'

$$H = y\, p_x - x\, p_y \, . \tag{5.2}$$

This, of course, is not bounded below, the price to be paid for writing a classical system quantum mechanically.

We may rewrite however,

$$\begin{aligned} H &= \tfrac{1}{4}(y+p_x)^2 - \tfrac{1}{4}(y-p_x)^2 - \tfrac{1}{4}(x+p_y)^2 + \tfrac{1}{4}(x-p_y)^2 \\ &= H_1 - H_2 \\ H_1 &= \tfrac{1}{2}P_1^2 + \tfrac{1}{2}Q_1^2 \ ; \qquad H_2 = \tfrac{1}{2}P_2^2 + \tfrac{1}{2}Q_2^2 \ ; \end{aligned} \tag{5.3}$$

with

$$\begin{aligned} P_1 &= \tfrac{1}{\sqrt{2}}\,(p_x + y) \ ; \qquad P_2 = \tfrac{1}{\sqrt{2}}\,(x + p_y) \ ; \\ Q_1 &= \tfrac{1}{\sqrt{2}}\,(x - p_y) \ ; \qquad Q_2 = \tfrac{1}{\sqrt{2}}\,(y - p_x) \ . \end{aligned} \tag{5.4}$$

The new variables P_i and Q_i obey the usual commutation rules:

$$[P_i,\, Q_j] = -i\delta_{ij} \ ; \qquad [P_i,\, P_j] = [Q_i,\, Q_j] = 0 \ , \tag{5.5}$$

so that the two parts of the hamiltonian (5.3) commute: $[H_1,\, H_2] = 0$. Thus, we found that although the classical harmonic oscillator has an unbounded quantum hamiltonian, we can decompose it into *two* genuine quantum hamiltonians, both of the familiar harmonic oscillator type and both bounded from below, but with a minus sign in between. All we now have to do is to 'postulate' that, say, $H_2|\psi\rangle = \frac{1}{2}\hbar\omega$ is imposed as a constraint equation on all states. Then we have a quantum harmonic oscillator.

This, however, is not the true solution to our problem. The splitting (5.3) is far too arbitrary. Where does the constraint $H_2 = \frac{1}{2}$ come from, and how can it survive interactions? How do we couple two or more such oscillators without spoiling the constraint? Let us analyze the reason why the splitting (5.3) is possible.

The classical harmonic oscillator has two conserved operators, besides the hamiltonian itself. These are:
— the radius r of the orbit: $\varrho^2 = x^2 + y^2$, and
— the *dilatation operator* $D = x\,p_x + y\,p_y - i$; classically, after all, the periodic solutions of the oscillator behave identically after a scale transformation $x,\, y \to \lambda x,\, \lambda y\,;\ p_x\,, p_y \to \lambda^{-1}p_x,\, \lambda^{-1}p_y$. We easily check that

$$[H,\, x^2 + y^2] = 0\ ; \qquad [H,\, D] = 0\ . \tag{5.6}$$

It is due to these operators that we can construct H_1 and H_2:

$$\begin{aligned} H_1 &= \frac{1}{4\varrho^2}(\varrho^2 + H)^2 + \frac{1}{4\varrho^2}(D+i)^2\ ; \\ H_2 &= \frac{1}{4\varrho^2}(\varrho^2 - H)^2 + \frac{1}{4\varrho^2}(D+i)^2\ . \end{aligned} \tag{5.7}$$

We see immediately that H_1 and H_2 commute, since D and ϱ commute with H (they do not commute with each other, but that does no harm), and we see that $H = H_1 - H_2$.

But we also see that, in this notation, the construction is fairly arbitrary. The contribution of the dilatation operator is unnecessary. We may perform the more interesting splitting:

$$\begin{aligned} \text{if} \qquad [\varrho,\, H] = 0 \qquad \text{then} \qquad & H = H_1 - H_2 \\ & H_1 = \frac{1}{4\varrho^2}(\varrho^2 + H)^2\ ; \\ & H_2 = \frac{1}{4\varrho^2}(\varrho^2 - H)^2\ . \end{aligned} \tag{5.8}$$

Now this generalizes to any function ϱ^2 of the coordinates x and y that happens to be conserved, $[\varrho^2,\, H] = 0$. So, in general, let

$$H = \mathbf{p}\cdot\mathbf{f}(\mathbf{q})\ ; \qquad [H,\, \varrho^2(\mathbf{q})] = 0\ , \tag{5.9}$$

and

$$\begin{aligned} H &= H_1 - H_2\ ; \\ H_{1,2} &= \frac{1}{4\varrho^2}(\varrho^2 \pm H)^2\ ; \qquad [H_1,\, H_2] = 0\ . \end{aligned} \tag{5.10}$$

Then introduce as a constraint:

$$H_2|\psi\rangle \to 0\ , \tag{5.11}$$

this implies that

$$H \to H_1 \to \varrho^2 \geq 0\,. \tag{5.12}$$

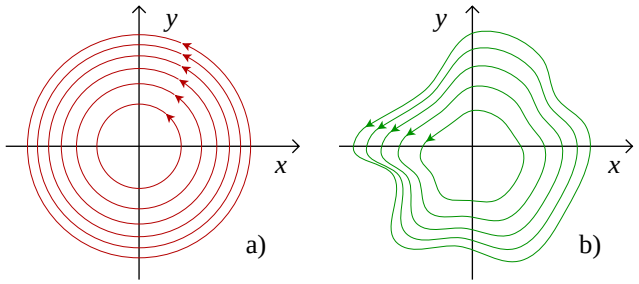


Fig. 3. Stable orbits, a) for the harmonic oscillator, b) an anhormonic oscillator. After switching on a dissipative term, the regions in between these trajectories will have only non-periodic solutions, tending towards the stable attractors.

The subclass of states obeying the constraint (5.11) obeys the Schrödinger equation $\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = -iH_1|\psi\rangle$, with a hamiltonian H_1 bounded from below. *But now we can motivate the constraint by introducing information loss!* This goes as follows. Let us assume that, in a deterministic system, there are stable orbits, separated by regions where solutions are non-periodic, attracted to the stable attractors. First, we mimic the system in terms of a non-dissipative model, where all orbits would be stable. it is then described by Eq. (5.9). Let the periods of these orbits be functions $T(\varrho)$ of ϱ. The constraint (5.11), (5.12) implies $H = \varrho^2$, or

$$e^{-iHT}|\psi\rangle = |\psi\rangle \ . \tag{5.13}$$

These are the states for which the periods $T(\varrho)$ obey

$$\varrho^2 T(\varrho) = 2\pi n \ , \qquad n \in \mathbb{Z} \ . \tag{5.14}$$

Thus, the constraint appears to correspond to limiting oneself to the stable orbits only. Note that, with Eq. (5.14), the hamiltonian can obtain any kind of eigenvalue spectrum, as opposed to the equidistant lines of the harmonic oscillator.

6. Conclusions

In this lecture we investigated *classical, deterministic, dissipative* models, and we found that, in general, they develop distinct stable orbits. The mathematics for analyzing these models requires that we first introduce non-dissipative equations, which allow a formalism using quantum mechanical notation, but, without dissipation, it cannot be understood why the hamiltonian would be bounded from below. Then we find that dissipation imposes constraints on the solutions, which appear to provide bounded hamiltonians. It is remarkable that dissipation also leads to an apparent *quantization* of the orbits, and this quantization indeed resembles the quantum structure seen in the real world.

The next step, yet to be taken, is to couple infinite numbers of dissipating oscillators to form models of quantum field theories. This may appear to be a very difficult task, but we do notice that in classical general relativity black hole formation is inevitable, and black holes indeed absorb information. This would imply that the distance scale at which dissipation plays a role must be the Planck scale.

At scales between the Standard Model and the Planck scale, the introduction of dissipation would be a new approach. Perhaps we can find models resembling Navier-Stokes liquids with viscosity. In non-relativistic models, the dimensionality of a viscosity η is given by

$$\big[\eta/\varrho\big] = [\mathrm{cm}^2/\mathrm{sec}] \, , \tag{6.1}$$

where ϱ is the mass density of the fluid. In a relativistic theory, where there is a fixed unit of velocity $c = 1$, the cm and the sec have the same dimensionality, so now η/ϱ has the dimensionality of a length. It is tempting to take this to be the Planck length. We may take strictly continuous fields, which however at distance scales ℓ small compared to the Planck scale are totally controlled by viscosity. There, at small Reynolds number [9],

$$R = \varrho u \ell/\eta\,, \tag{6.2}$$

where u are the typical velocities, the field distributions show no further structure, but at distance scales large compared to the Planck scale, one may expect 'turbulence', *i.e.*, chaotic behaviour, for which we propose the introduction of apparently quantum mechanical techniques in order to describe the statistics. One then may invoke the renormalization group in order to reach the length scales of the Standard Model, and make contact with the real world.

It is far too early to ask for tangible results and firm testable predictions of the approach that we have in mind. A very indirect prediction may perhaps be made. We conjecture that the apparently quantum mechanical nature of our world is due to the statistics of fluctuations that occur at the Planck scale, in terms of a regime of completely deterministic dynamics. This would entail that all quantum mechanical effects should be reproducible in some deterministic model, including all machinations with what is usually called a 'quantum computer'. As is well-known, quantum computers, if they can be constructed, will be able to do computations no ordinary computer can accomplish. This would be a contradiction with our claim that it can be mimicked using ordinary computers. However, we are unable to mimic any quantum system we like. Interactions are essential and unavoidable. It is also these interactions that cause unwanted decoherence in a quantum computer. Experimenters are trying to create devices in which the ideal situation is approached as well as possible. I now claim that it will be impossible to shield these devices from the unwanted interactions, so that the ideal quantum computer can never be built. More precisely:

> *No quantum computer can ever be built that can outperform a classical computer if the latter would have its components and processing speed scaled to Planck units.*

Because the Planck units are extremely tiny, this still leaves lots of room for quantum computers to do miracles, but eventually, there will be a limit. An ideal quantum computer that would consist of millions of parts, could, in principle, factor integers with millions of digits into prime numbers. It appears that present programs can factor a number of N digits using memories of the order of $10^{\sqrt{N \log N}}$ cells, in $10^{\sqrt{N \log N}}$ steps. Perhaps a reasonable computer takes 10^{120} Planck volumes; that would limit the factorizable numbers to 10^{4000} or so. Thus we predict that even a quantum computer will not be able to exceed such limits in practice.

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